

# The deconfinement transition in $SU(N)$ lattice gauge theories at large- $N$

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- Focus on  $4D$ , recent  $3D$  at large- $N$  see **Liddle and Teper '05-'06, Holland '05**.
  - Recent years activity, not older reduced models (but see **Das '87**).

## Outline of the talk

### I. Large- $N$ from the lattice :

- Background and lattice techniques.

### II. $T \simeq T_d$ - nature of the transition :

- Order of the transition.
- Deconfinement vs. Hagedorn.

### III. $T > T_d$ - Properties of the deconfined phase :

- Bulk thermodynamics.
- Debye masses and spatial string tension.
- Domain wall tension.
- Topology.
- For  $\chi S$ -restoration see [Narayanan's talk](#).

### IV. Summary

## I. Large $N$ from the lattice

't Hooft '72 : For  $SU(N)$ , take  $N \rightarrow \infty$  with  $g^2 N$  fixed  $\rightarrow$  expansion in  $1/N$ .

Witten '79 :  $N = \infty$  “planar” is a simpler theory:

- Free mesons and glueballs (no decays, mixings or scatterings).
- Classical, soliton-like, baryons ( $m_B \rightarrow \infty$ ).

Since  $N^2$  gluons  $\gg N$  quarks : QCD $_{\infty}$  = (quenched QCD) $_{\infty}$

$SU(\infty)$  still unsolved  $\rightarrow$  **lattice** :

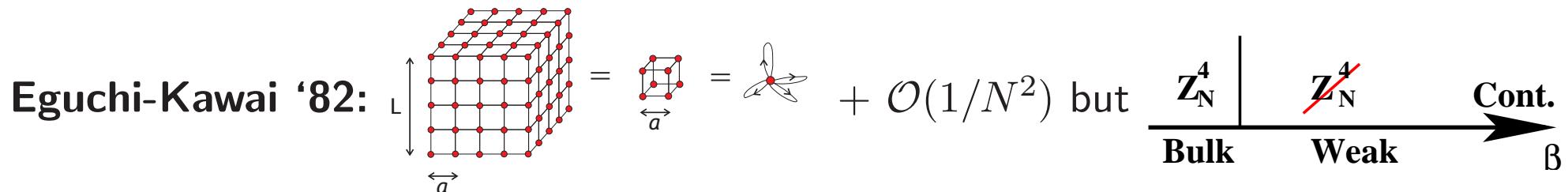
Define  $SU(N)$  on a 4D torus of  $L_s^3 \times L_t$ ,  $S(\text{Wilson})$ ,  $\beta = \frac{2N}{g^2} \sim N^2$ .

**Standard route of large- $N$  from the lattice** : Do  $SU(2, 3, 4, 6, 8, \dots)$

$$\langle O(N) \rangle = \langle O(\infty) \rangle + \frac{a}{N^2} + \frac{b}{N^4} \dots \Leftrightarrow \text{get limit + corrections.}$$

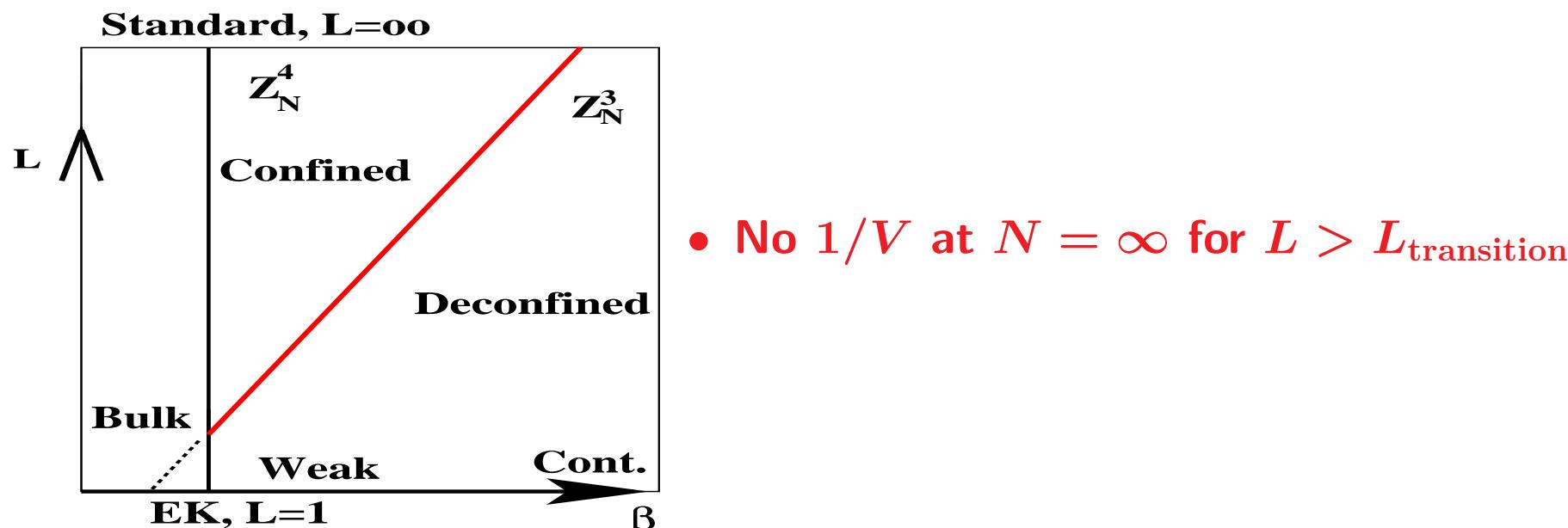
Teper and collaborators '98-'06, Del Debbio, Panagoupolous and Vicari '02,'06, de Forcrand, Lucini and co. '04,'05

## Less standard route of large- $N$ from the lattice : reduction



- Fails in continuum **Bhanot, Heller and Neuberger '82** → Twisted,Quenched EK ('80) (**Das and Kogut, Fabricuis and Haan, Gocksch and Neri, Gonzalez-Arroyo and Okawa, Klinkhamer, Neuberger, Rossi**).

A variant of EK reduction : partially reduce  $\infty^4 \rightarrow L^4 > 1^4$  **Neuberger et al '02-'06**.



## II. $T \simeq T_d$ - nature of the transition

**The Ultimate temperature idea** by **Hagedorn '65** - pre-dated QCD

$pp$  particle multiplicities → hadrons consistent only  $T < T_H \simeq 158$  MeV.

**Interpret  $T_H$  as 2nd-order transition of quark liberation** by **Cabibo and Parisi '75**.

**Heuristic description for Hagedorn/deconfinement** given by **Banks and Rabinovici '79**:

Confinement means

$$\left. \begin{array}{l} E(l) = \sigma l \\ \rho(l) = \exp(+cl) \end{array} \right\} \rightarrow Z(T) = \sum_l \rho(l) e^{-E(l)/T} = \sum_l e^{(c-\sigma/T)l}.$$

So  $Z(T = \frac{\sigma}{c} \sim \sqrt{\sigma}) \rightarrow \infty$  and long loops proliferate with  $m(T = T_H \equiv \frac{\sigma}{c}) = 0$ .

**These arguments** lead to 2nd order, but ignore interactions between (string-like)Hadrons.

**Natural at  $N = \infty$**   $\Rightarrow$  Is the transition 2nd order at  $N = \infty$  ?

**Clash with reduction :**

- $\langle W \rangle$  are independent of  $L$  for  $L > L_{\text{transition}}$  **recent Neuberger et al.**

then relabel axes and

- $\langle W \rangle$  are independent of  $T$  for  $T < T_{\text{transition}}$  **Gocksch and Neri '83**

If  $T_{\text{transition}} = T_d$  then choose  $\langle W \rangle = \langle \square \rangle \sim \text{Entropy}$ ,

$\text{Entropy}(T_d^-) = \text{Entropy}(0) \sim \mathcal{O}(N^0)$ , but  $\text{Entropy}(T_d^+) \sim \mathcal{O}(N^2)$

**So what happens on the lattice ?**

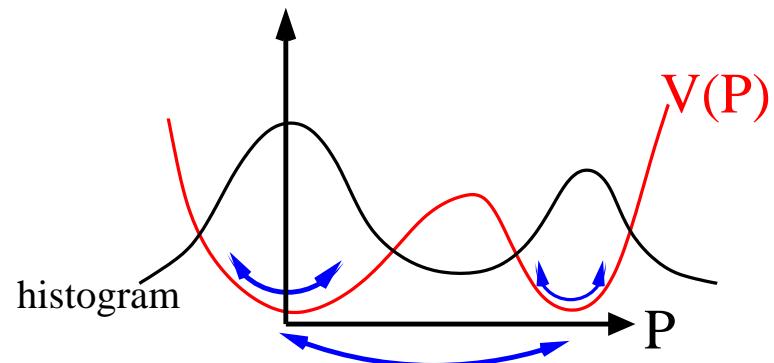
**Numerically** :  $SU(2)$  is 2nd while  $SU(3)$  is weakly 1st.

McLaren and Svetitsky '81,  
Fukigita et al. '89, Brown et  
al. '88

**Old  $SU(4)$**  : 1st order but near bulk, not continuum ( $L_t \leq 4$ ).

Svetitsky and Batrouni,  
Gocksch and Okawa,  
Gross and Wheater '83

**Confirmed** by Wingate and Ohta '99 ( $S_W, < 20^3 6$ ) **and Gavai '02** ( $S_W^+, L_t = 4 - 8, L_s \geq 2L_t$ ) , :



Signs of first order :

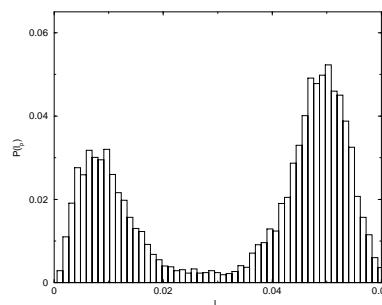
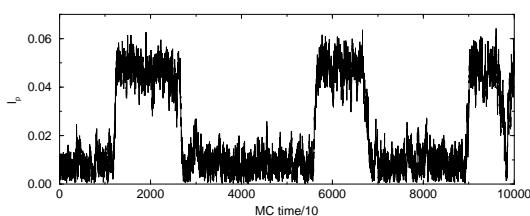
- Double peak structure in histogram of  $\langle P \rangle$ .
- Tunneling configurations.
- Finite size scaling:  $\langle P^2 \rangle - \langle P \rangle^2 \sim V$ .

**The latent heat :**

$$L_h = \Delta\epsilon, \quad \frac{\epsilon}{T^4} = \frac{1}{T^2 V} \frac{\partial \log Z}{\partial T} = \frac{1}{T^2 V} \frac{\partial \log Z}{\partial \beta} \times \frac{\partial \beta}{\partial T} = L_t^4 \langle \square \rangle \times \frac{\partial \beta}{\partial \log a^{-1}}.$$

Find  $L_h = 0.67(2) \times SB$  with perturbative scaling **Higher than  $SU(3)$  !!!**

# Systematic study of $SU(2 \leq N \leq 8)$ Lucini, Teper and Wenger '02-'05



At  $\beta_d(L_t, L_s)$ :

$$\chi(P) = \langle P^2 \rangle - \langle P \rangle^2 = \text{max.}$$

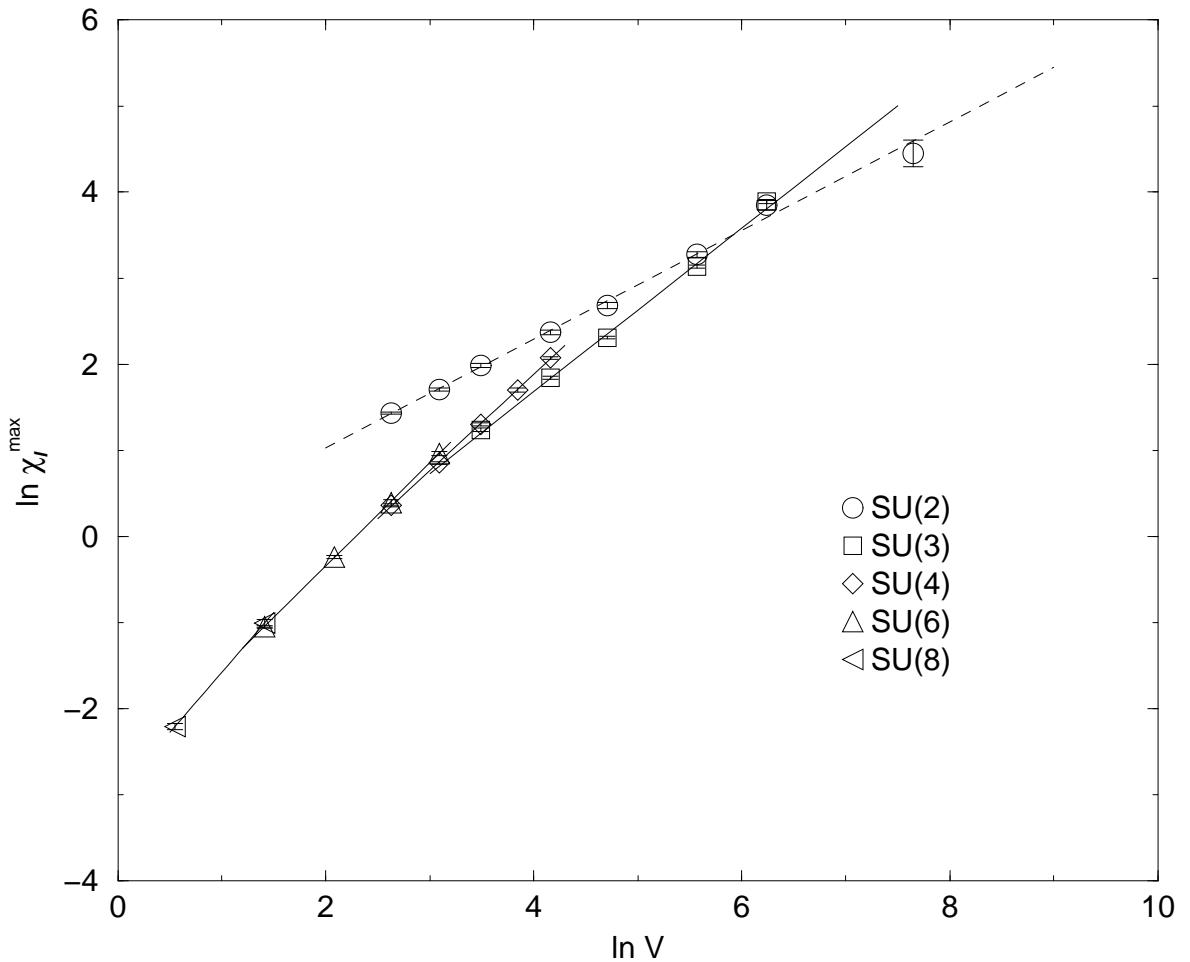
At  $\beta_d(L_t, L_s)$ :  $\chi_{\text{max}} \sim V^\gamma$ :

$$\gamma = \begin{cases} 1 & 1st \\ < 1 & 2nd \end{cases}$$

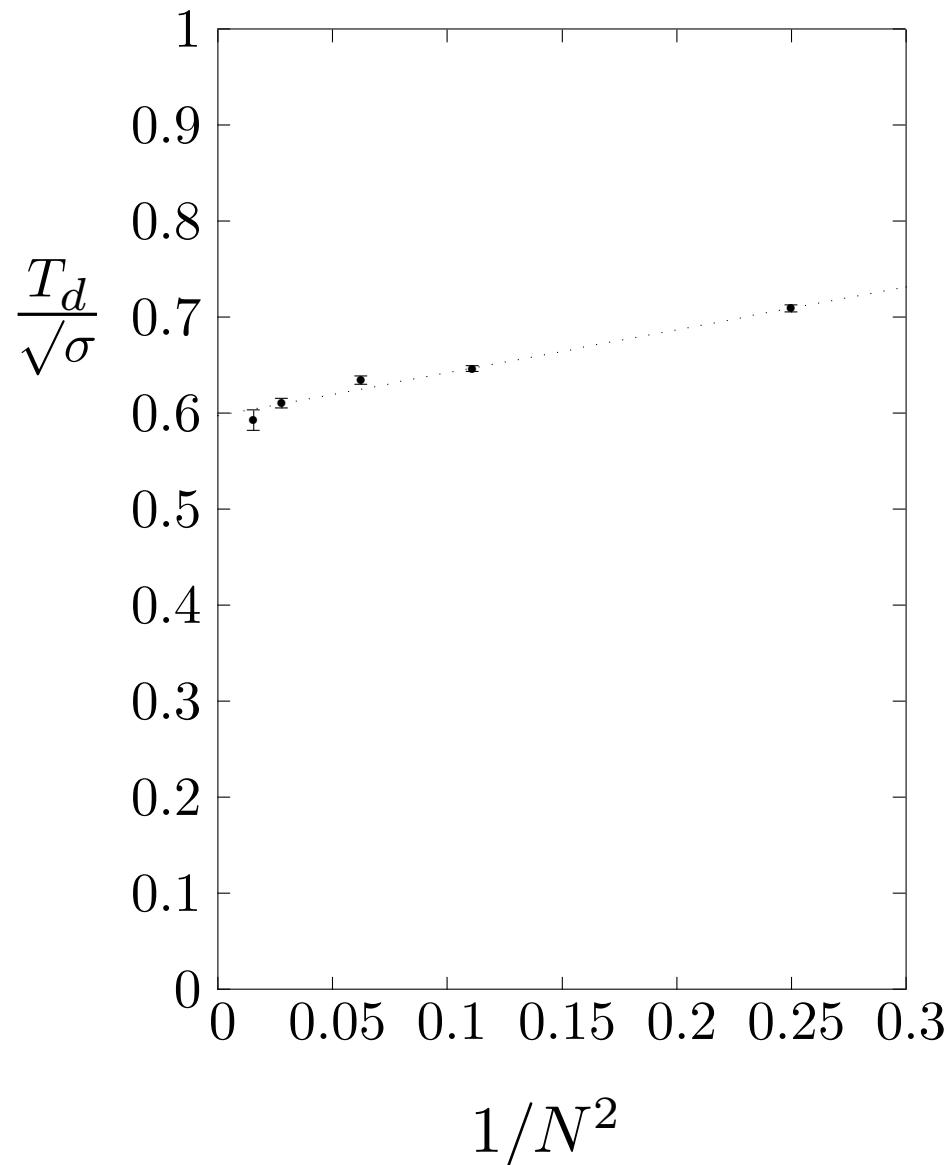
$$N = 4 : (12 - 20)^3 5, 16^3 6, 24^3 8$$

$$N = 6 : (8 - 14)^3 5, 16^3 6, 16^3 8$$

$$N = 8 : (8 - 10)^3 5, 8^3 6, (10 - 12)^3 8$$



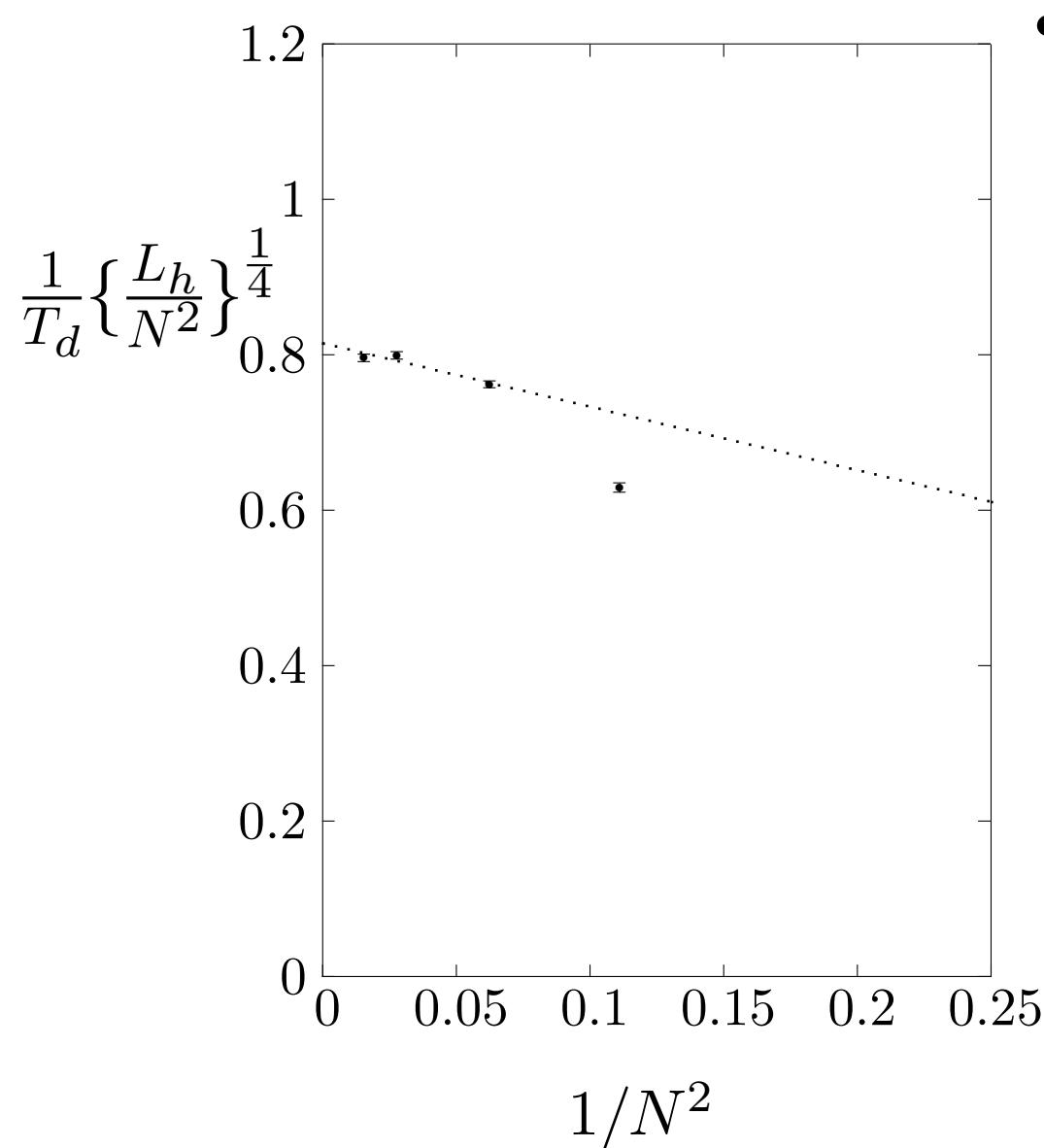
And after a  $V \rightarrow \infty, a \rightarrow 0$  get : [use Non-perturbative scaling of  $a\sqrt{\sigma}(\beta)$ ]



- $T_d(3) - T_d(\infty) \simeq 8\%$
- $\frac{T_d}{\sqrt{\sigma}}(\infty) = 0.5970(38)$
- Neuberger's et al.:  $\frac{L_d^{-1}}{\sqrt{\sigma}}(\infty) \simeq 0.64$ .  
 $5^4 - 10^4$  and  $SU(41 - 23)$
- $V \rightarrow \infty$  extrapolation:

$$T_d(V) = T_d(\infty) + \frac{\mathcal{O}(1/N^2)}{V}$$

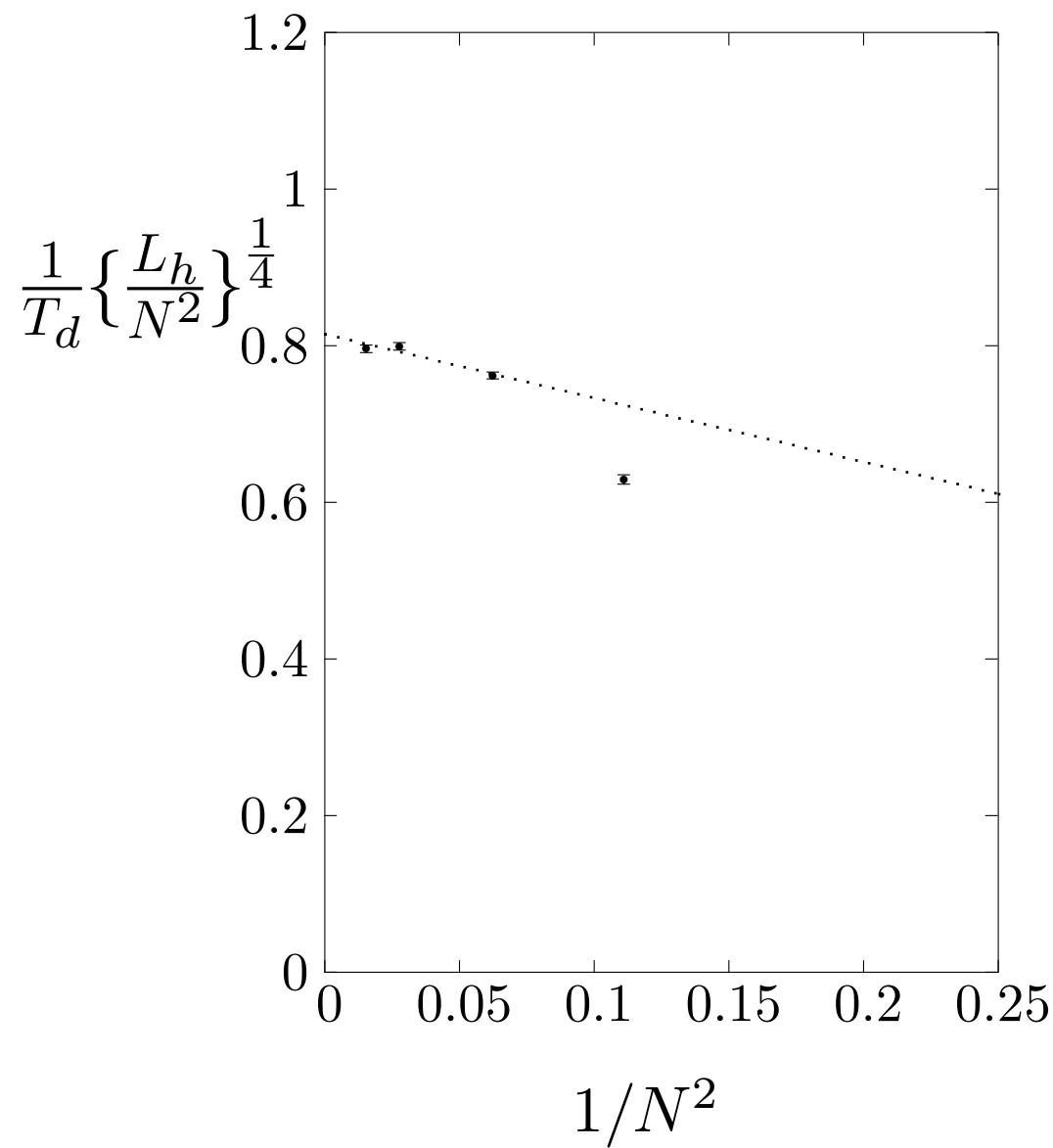
**And after a  $V \rightarrow \infty$  but at  $a^{-1} = 5T_d$  get :  $SU(4, 6, 8) :$   $< 32^3 5, 16^3 5, 12^3 5$**



- After  $a \rightarrow 0$  extrapolation:  

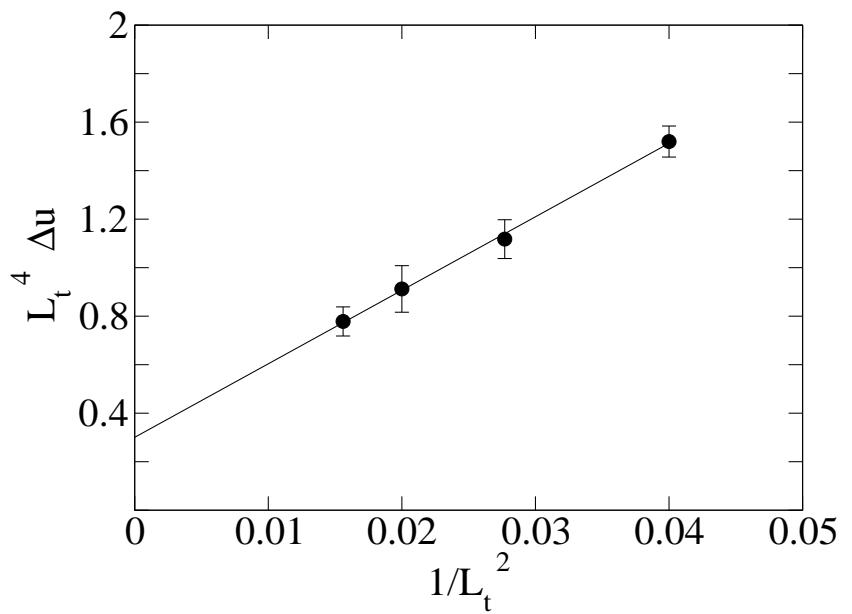
$$\frac{1}{T_d} \left( \frac{L_h}{N^2} \right)^{1/4} = 0.766(40)$$
or  
 $L_h / S.B. = 0.52(11)$

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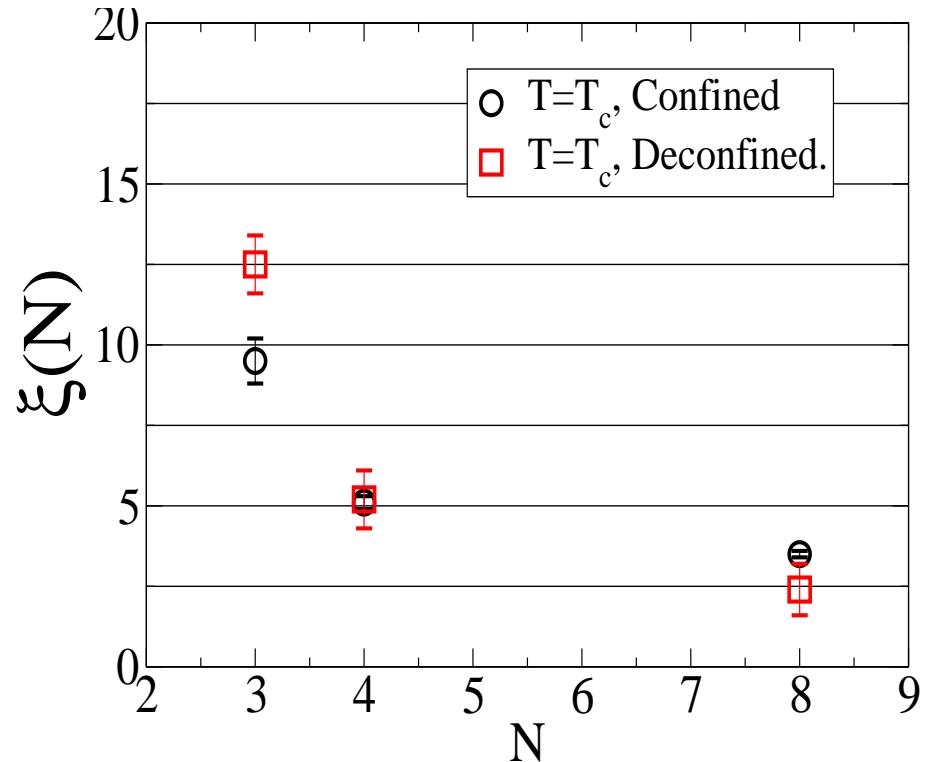
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$$\frac{1}{T_d} \left( \frac{L_h}{N^2} \right)^{1/4} = 0.766(40)$$
or  
 $L_h/S.B. = 0.52(11)$
- Kiskis '05 :  $N = 29, 37, V = 5^4 - 8^4$



$L_h/S.B. \simeq 0.26$  (scaling ?)

From  $\langle P_x P_{x+R} \rangle - \langle P_x \rangle \langle P_{x+R} \rangle \sim e^{-m_t R}$  see that at  $T_d$ :



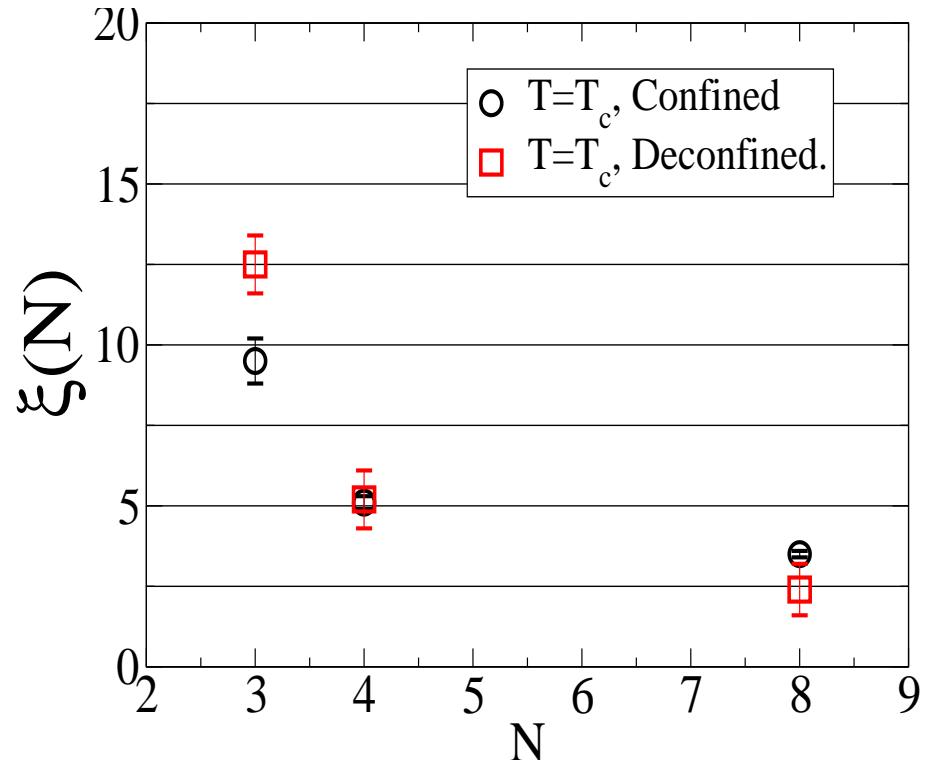
$$\begin{array}{cccc} SU(3) & SU(4) & SU(6) & SU(8) \\ 64^3 5 & < 32^3 5 & 16^3 5 & 12^3 5 \end{array}$$

Again: expect smaller  $1/V$  terms.

Possible because suppressed tunnellings:

$$P(T_d) \propto \exp \{-2\sigma_{cd} A/T_d\} \sim \exp \{-0.03 \times (NT_d L)^2\} \sim 10^{-11} \text{ for } SU(12), 12^3 5$$

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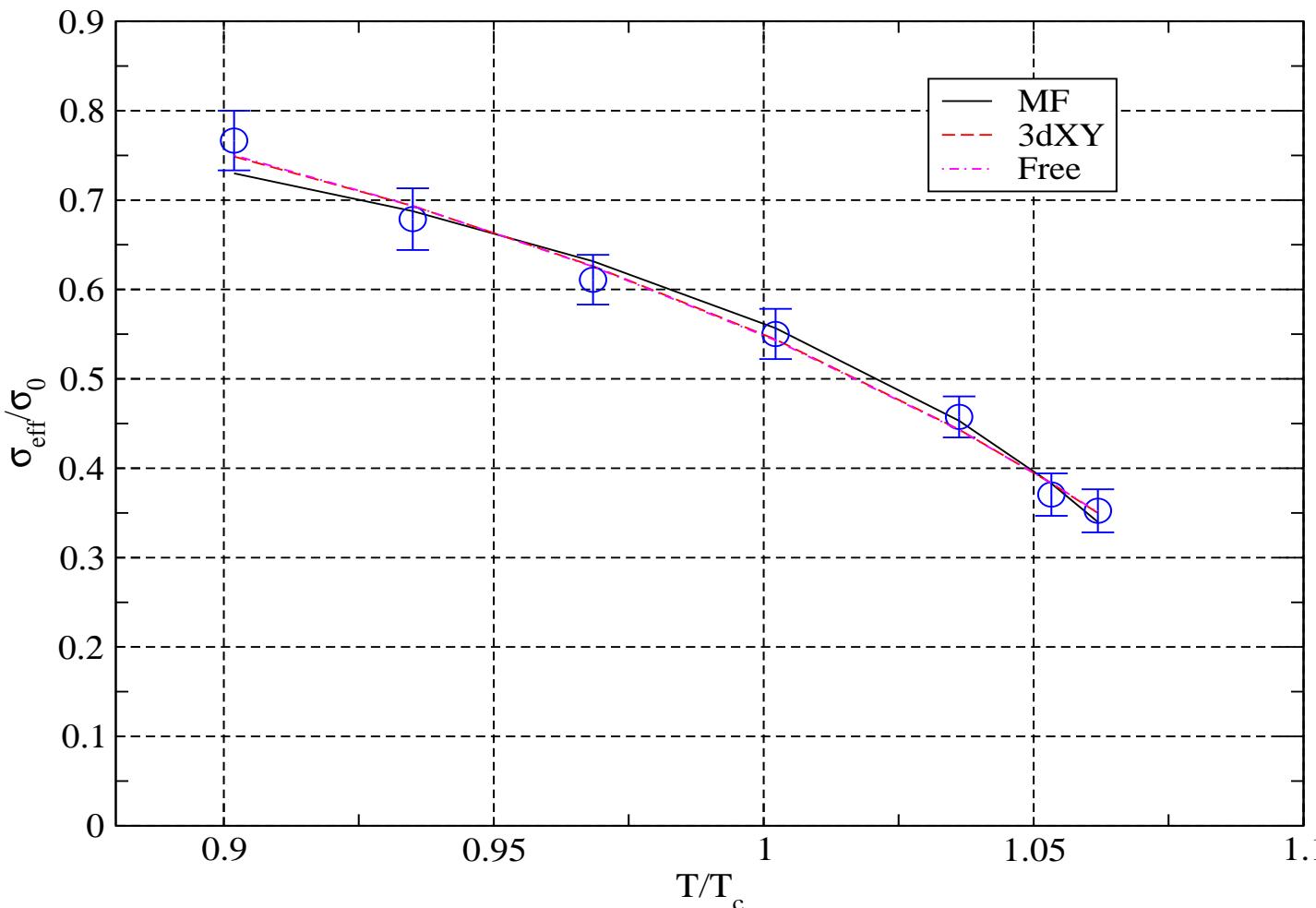
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All evidence → deconfinement is 1st order at  $N \rightarrow \infty$

**Where is Hagedorn's  $T_H$  ?** Deconfinement ‘protects’ from divergences at  $T_H > T_d$ .

**Numerical MC’s** : Look at Polyakov-loop mass  $m_t(T > T_d)$ . **BB and Teper ‘05**

- $m_t(T)$  decreases with  $T$ , extrapolate to  $m_t(T^*) = 0$ , and identify  $T^* = T_H$ .
- Do  $SU(8, 10, 12)$  on  $12^3 5$  (tunneling  $\sim 10^{-5} - 10^{-11}$ ), and find



- $\sigma_{\text{eff}} = m_t/L_0 \equiv m_t \cdot T$
- $T_H/T_d = 1.092(6)$ ,  
 $\nu = MF$
- $T_H/T_d = 1.116(9)$ ,  
 $\nu = 3dXY$
- $T_H/T_d \simeq 1.1$

### III. $T > T_d$ - Properties of the deconfined phase :

What replaces Hadronic phase after deconfinement ?

**Recent years** : evidences that for  $T_d < T \lesssim 2T_d$ , simple QGP is unsuitable.

Lattice : 10 – 20% deviation from free gas up to  $\sim 4T_d$ . Boyd et al. '96

Wish to compare other observables to perturbation theory :

- Mass gaps (Debye masses, spatial string tensions).
- Domain wall interface tensions.
- Instantons density.

Large- $N$  can help since

- Perturbation theory becomes simpler (only planar diagrams).
- Physical models should become simpler.

### III.A. Bulk thermodynamics on the lattice - BB and Teper '05

The integral method : Boyd et al. '95, '96

$$P(T) = \frac{T}{V} \log Z = \frac{1}{a^4(\beta) L_s^3 L_t} \int_{\beta_0}^{\beta} d\beta' \underbrace{\frac{\partial \log Z}{\partial \beta'}}_{6L_t L_s^3 \langle \square \rangle_{\beta'}} = \frac{6}{a^4(\beta)} \int_{\beta_0}^{\beta} d\beta' \langle \square \rangle_{\beta'} + [P(T_0)]$$

Regularization :  $P(T, \beta) \rightarrow P(T, \beta) - P(0, \beta)$  so  $\langle \square \rangle \rightarrow \delta \langle \square \rangle$ .

Lattice volumes :  $16^3 5$  for  $SU(4)$ ,  $8^3 5$  for  $SU(8)$ ,  $20^3 5$  for  $SU(3)$

Asymmetric lattices  $T > 0$ :  $\rightarrow SU(8)$ :  $\delta \langle \square \rangle \sim 1\sigma$  from  $L_s = 8, 14$ .

Is  $\langle \square \rangle_{T_d^-} = \langle \square \rangle_0$  ? :

- $SU(3)$ :  $15\sigma$  X data from Boyd et al. '96
- $SU(4)$ :  $1.7\sigma$  much better.
- $SU(8)$ :  $2.1\sigma$  much better.

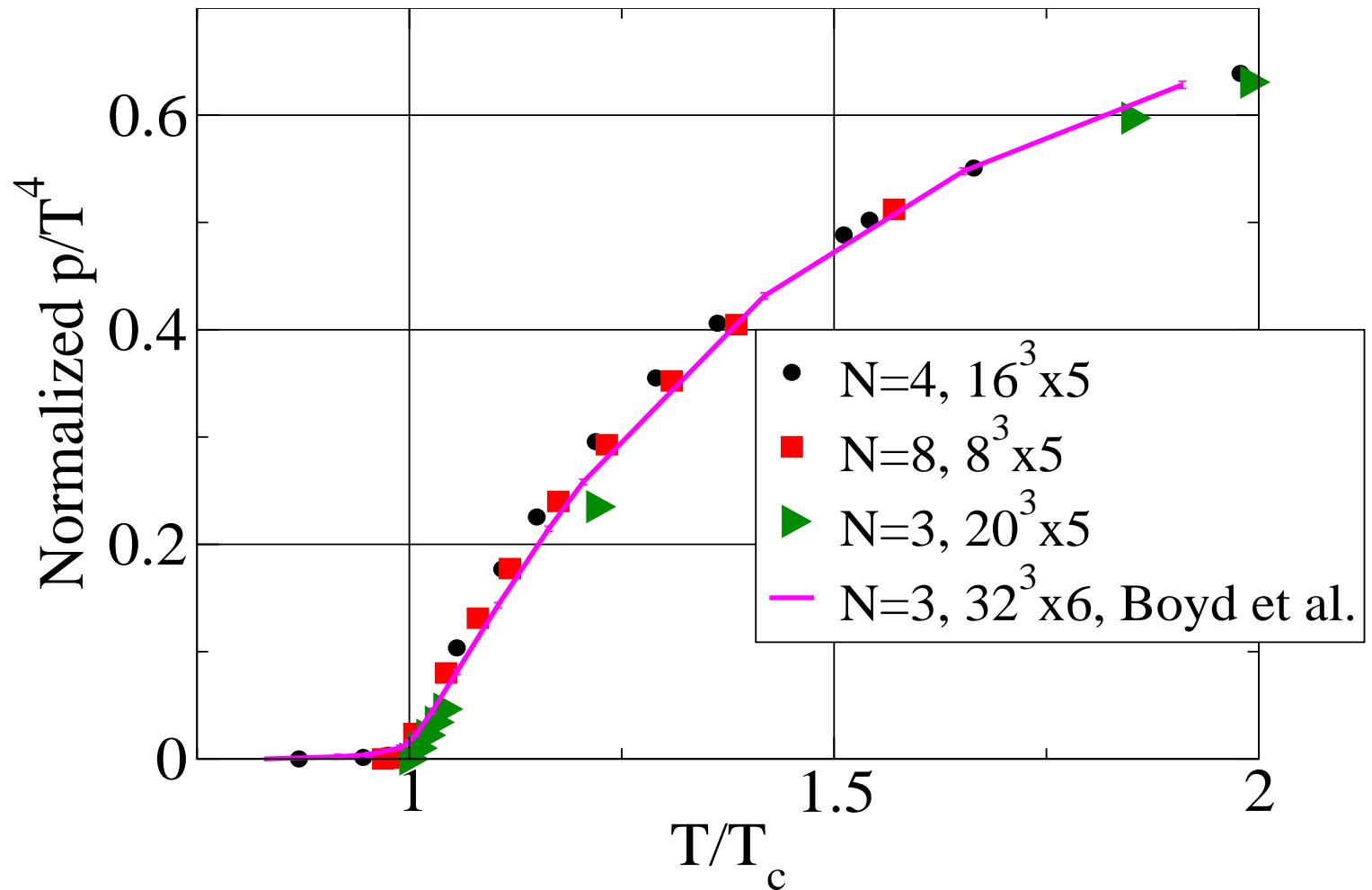
So  $[P/T^4]_{T < T_d} \simeq 0$  for larger  $N \rightarrow$  small systematics from integration constant.

### III.A. Pressure normalized to lattice Stephan Boltzman ( $\sim N^2$ )

$\frac{p/T^4}{\text{free}}$ , free =  $(N^2 - 1) \frac{\pi^2}{45} [1 + \mathcal{O}(1/L_t^2)]$  Boyd et al. '96, Heller and Karsch '84.

↓  
1

Pressure plots lie almost on top of each other.

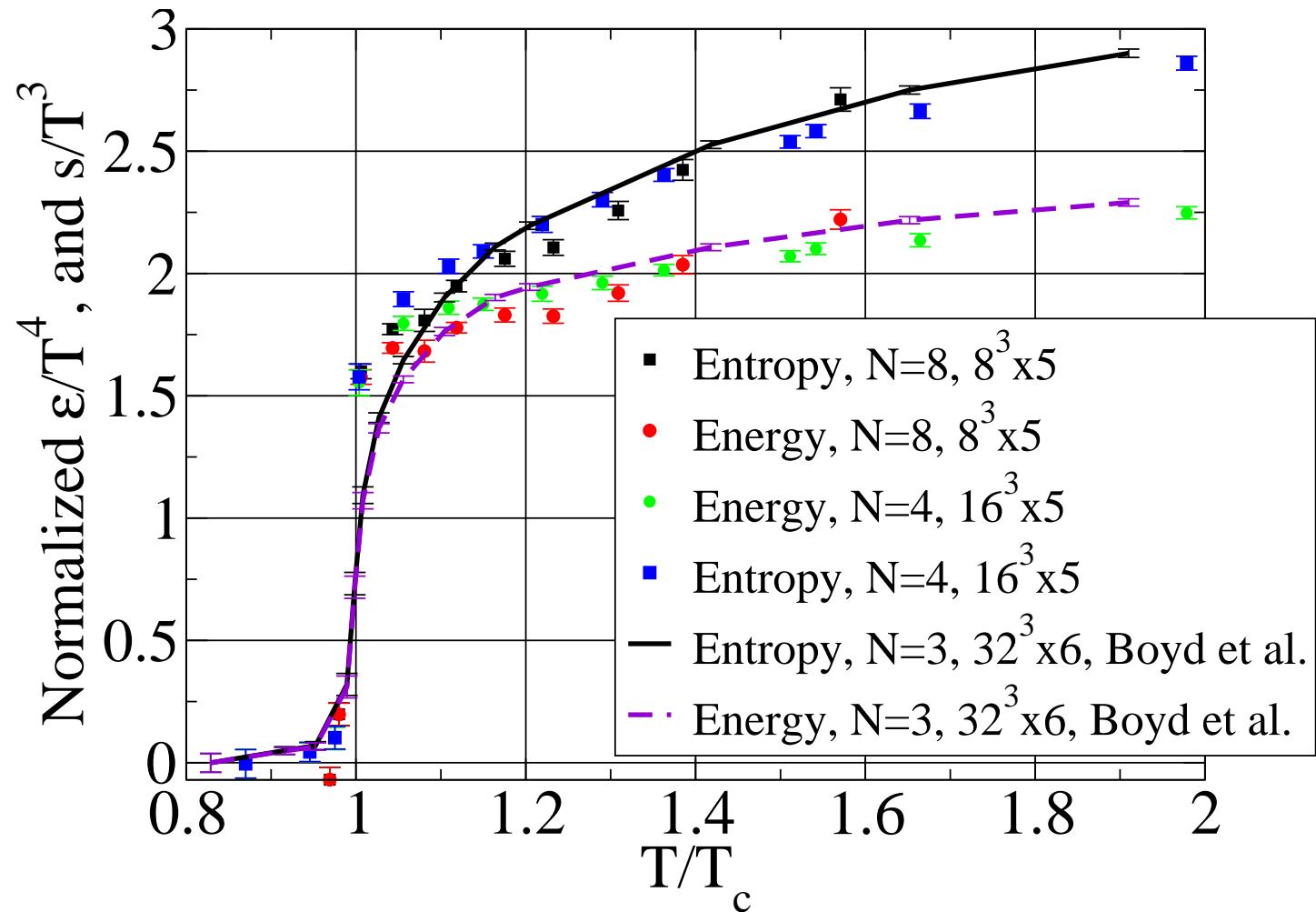


### III.A. Energy and entropy normalized to lattice Stephan Boltzman ( $\sim N^2$ )

$$\frac{s/T^3}{\text{free}}, \frac{\epsilon/T^4}{\text{free}}, \text{ free} = (N^2 - 1) \frac{\pi^2}{45} [1 + \mathcal{O}(1/L_t^2)] \quad \text{Boyd et al. '96, Heller and Karsch '84.}$$

$\downarrow 4$        $\downarrow 3$

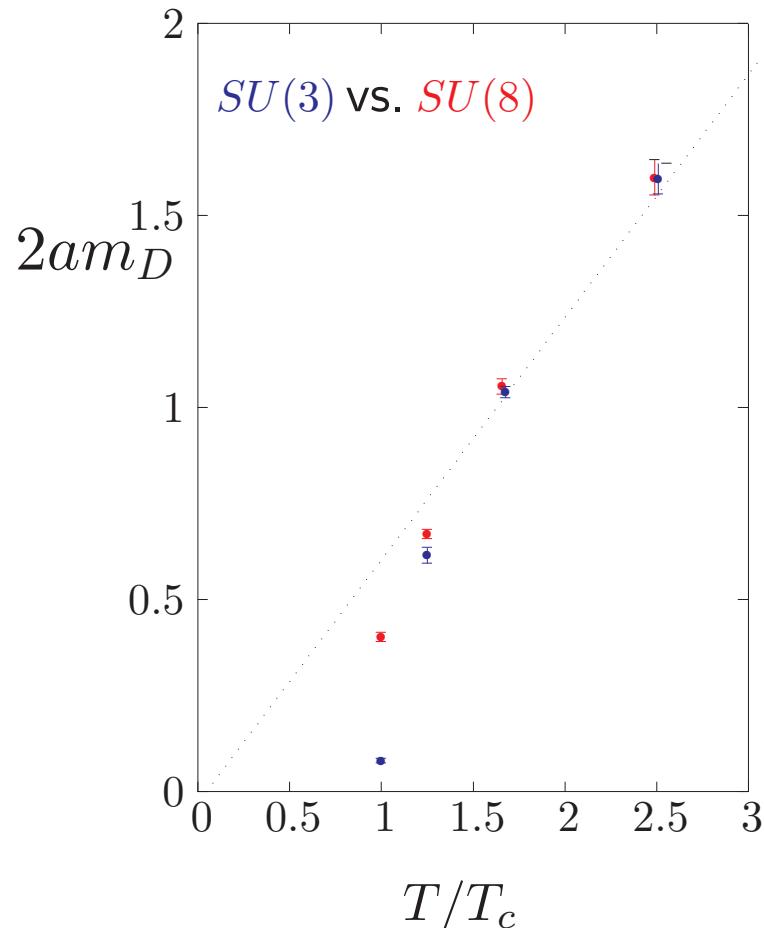
$s/T^3, \epsilon/T^4$  have  
 modest  $O(1/N)$   
 corrections.



### III.B. Lattice vs. perturbation theory - Debye screening

Lucini, Teper and Wenger '05

$$\langle P_x P_{x+R} \rangle - \langle P_x \rangle \langle P_{x+R} \rangle \sim \langle A_0^2(x) A_0^2(x+R) \rangle \sim e^{-2m_D R} \quad ; \quad m_D = T \sqrt{\frac{g^2(T)N}{3}}$$



- $m_D \propto T$ , independent of  $N$  for  $T \geq 1.6T_d$ .
- $\frac{g^2 N(1.6T_d)}{4\pi} \simeq 0.65$ .

$$T = \frac{5T_d}{(2-4)} \text{ from } 8^3(2-4).$$

### III.B. Lattice vs. perturbations : spatial masses $\langle P_x^s P_{x+R}^s \rangle \sim e^{-m_s R}$

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- $a\sqrt{\sigma}(T = 0) = 0.346(2)$   
 $a\sqrt{\sigma}(T = T_d^-) = 0.349(5).$
- For  $SU(8)$  see that  
 $m_s(T_d^-) - m_s(T_d^+) \simeq 15\%.$

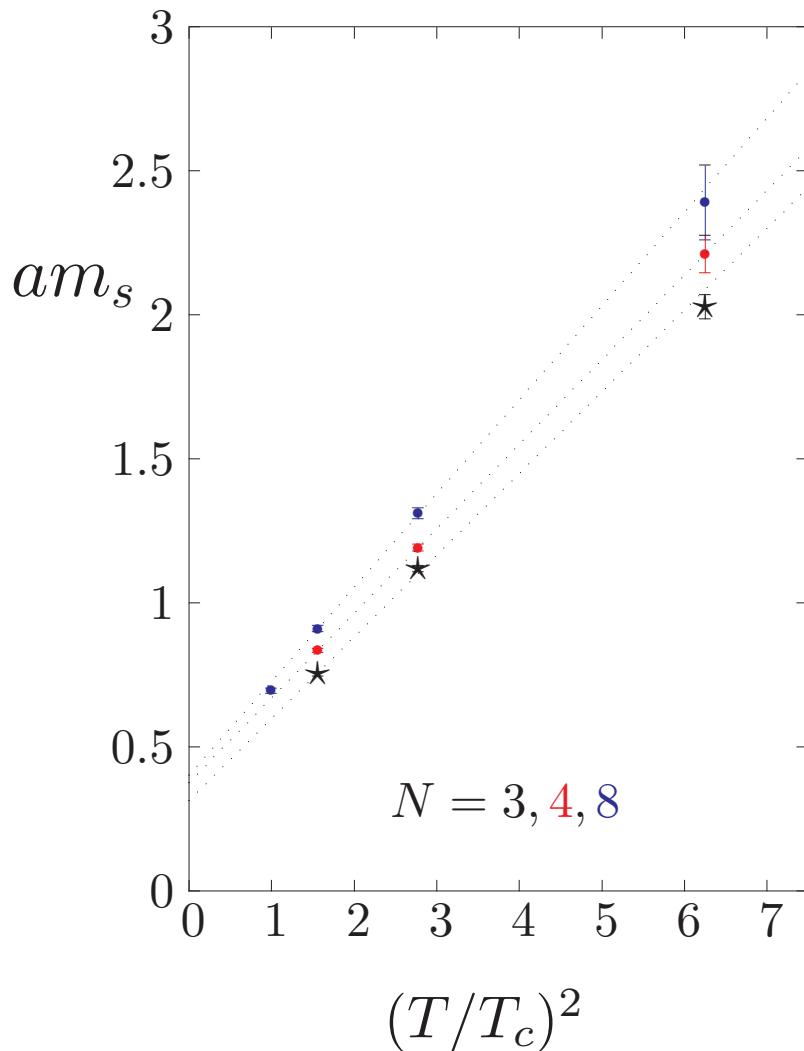
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- For  $SU(8)$  see that

$$m_s(T_d^-) - m_s(T_d^+) \simeq 15\%.$$

- “Early” large- $T$  behavior.

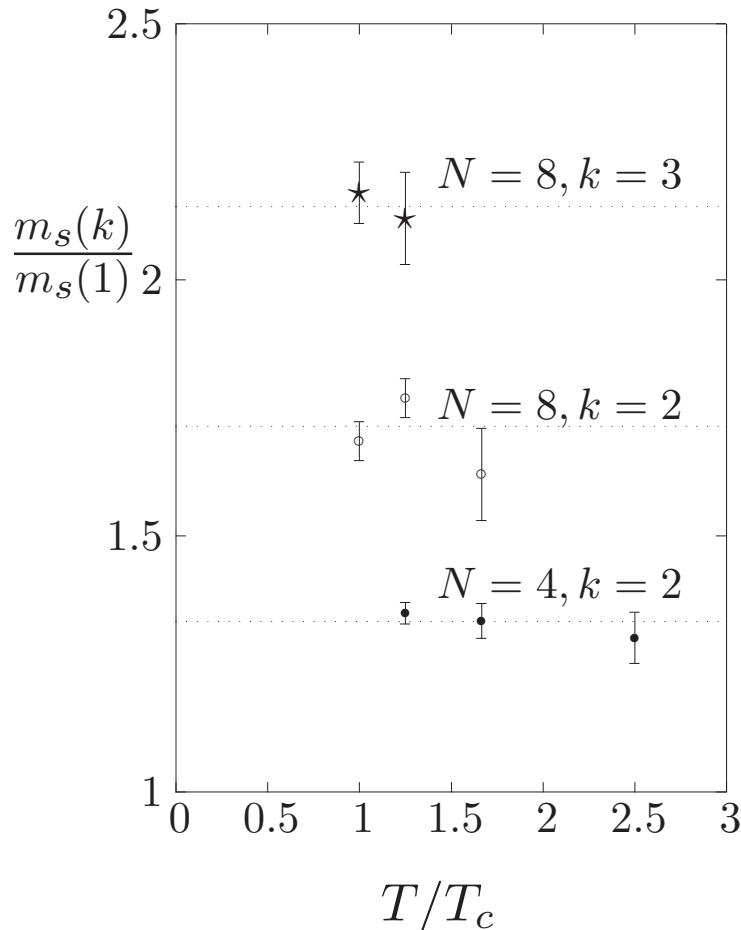


For  $T \gg T_d$  get **2 + 1** with  $g_3^2 N \sim g^2 N T$  and  $\sigma_s(3D) = (g_3^2 N)^2 \sim (g^2 N T)^2$

### III.B. Lattice vs. perturbations : spatial masses $\langle P_x^s P_{x+R}^s \rangle \sim e^{-m_s R}$

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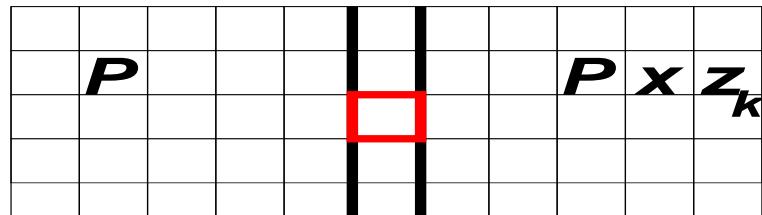
Look at  $k$ -strings : bound states of  $k$  fundamental strings, stable for  $N > 3$ .



- Casimir scaling strongly preferred over “sine” law.
- Similar to 3D rather than 4D
  - “Early” dimensional reduction ?
- Will be good to compare with perturbation theory à la **Schroder and Laine ‘05**.

## III.B. Lattice vs. perturbations : Domain wall tensions

Above  $T_d$  :  $N$  equivalent g.s.'s separated by domain walls and



$$\frac{\mathcal{Z}_k(A)}{\mathcal{Z}_0(0)} = e^{-F_k(A)/T} = e^{-\Sigma_k \cdot A/T}$$

Perturbation theory : Korthals Altes and collaborators '92,'01,'04

$$\Sigma_k = k(N - k) \frac{4\pi^2}{3\sqrt{3}} \frac{T^3}{\sqrt{g^2(T)N}} + O(g)$$

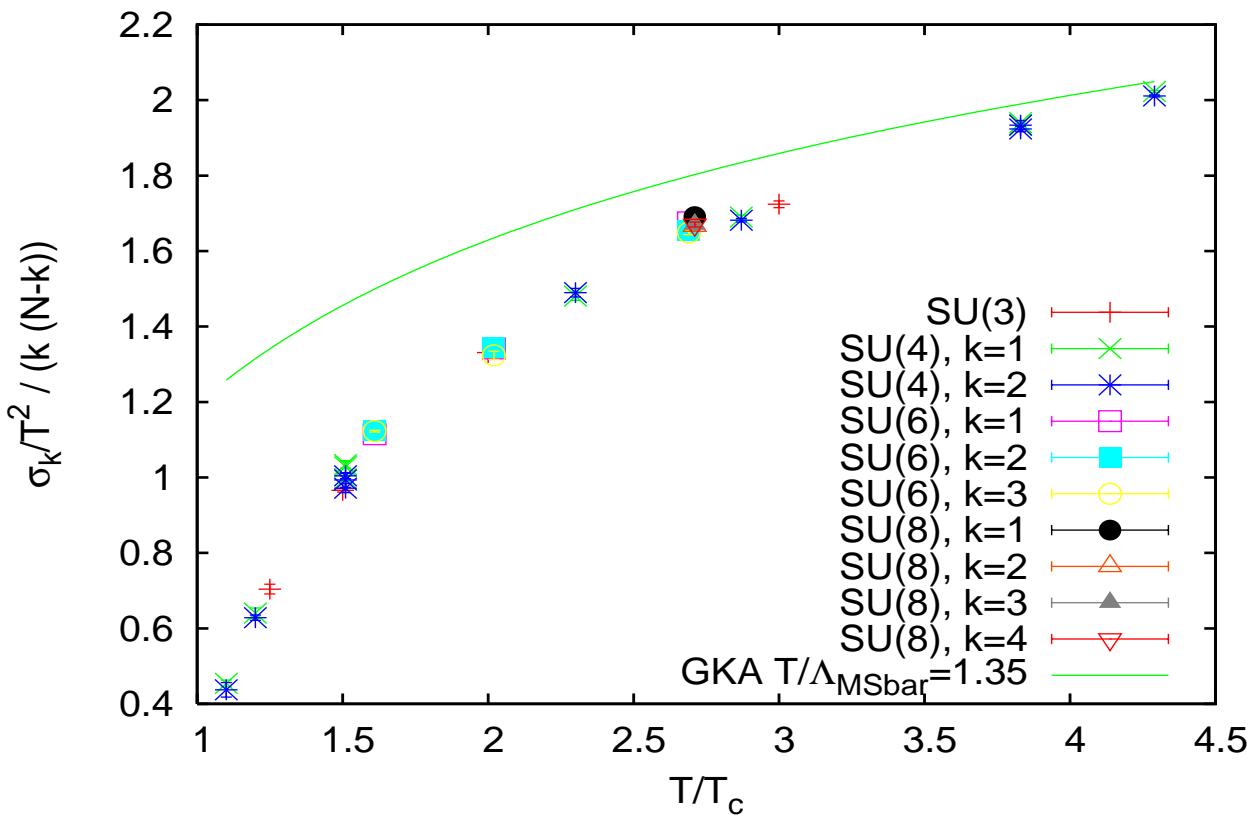
To get  $\Sigma_k$ :

- Enforce a wall by  $\beta_{\square} \rightarrow e^{i \frac{2\pi k}{N}} \beta_{\square}$ .
- Measure  $\mathcal{Z}_k(A)/\mathcal{Z}_0(0) = \langle e^{-S(\text{flipped})} \rangle_0 \rightarrow$  overlap problem.

Two ways out :

- $\partial_\beta \left( \frac{\Sigma_k A}{T} \right) = \partial_\beta \left( \frac{F_k}{T} \right) = \partial_\beta (\log Z_0 - \log Z_k) = \langle S_k \rangle_k - \langle S_0 \rangle_0$  Bursa and Teper '05
- $\frac{Z_k(A)}{Z_k(0)} = \frac{Z_k(A)}{Z_k(A-1)} \cdot \frac{Z_k(A-1)}{Z_k(A-2)} \cdots \frac{Z_k(1)}{Z_k(0)} = \prod_{i=1}^A e^{-\Sigma_k a^2/T}$  de Forcrand et al. '04-'05

Results : lattice vs. perturbation theory.



de Forcrand, Lucini and Noth '05

$SU(2, 3, 4, 6, 8), T/T_d \geq 1.1. L_t \geq 5, L_s \geq 3L_t$

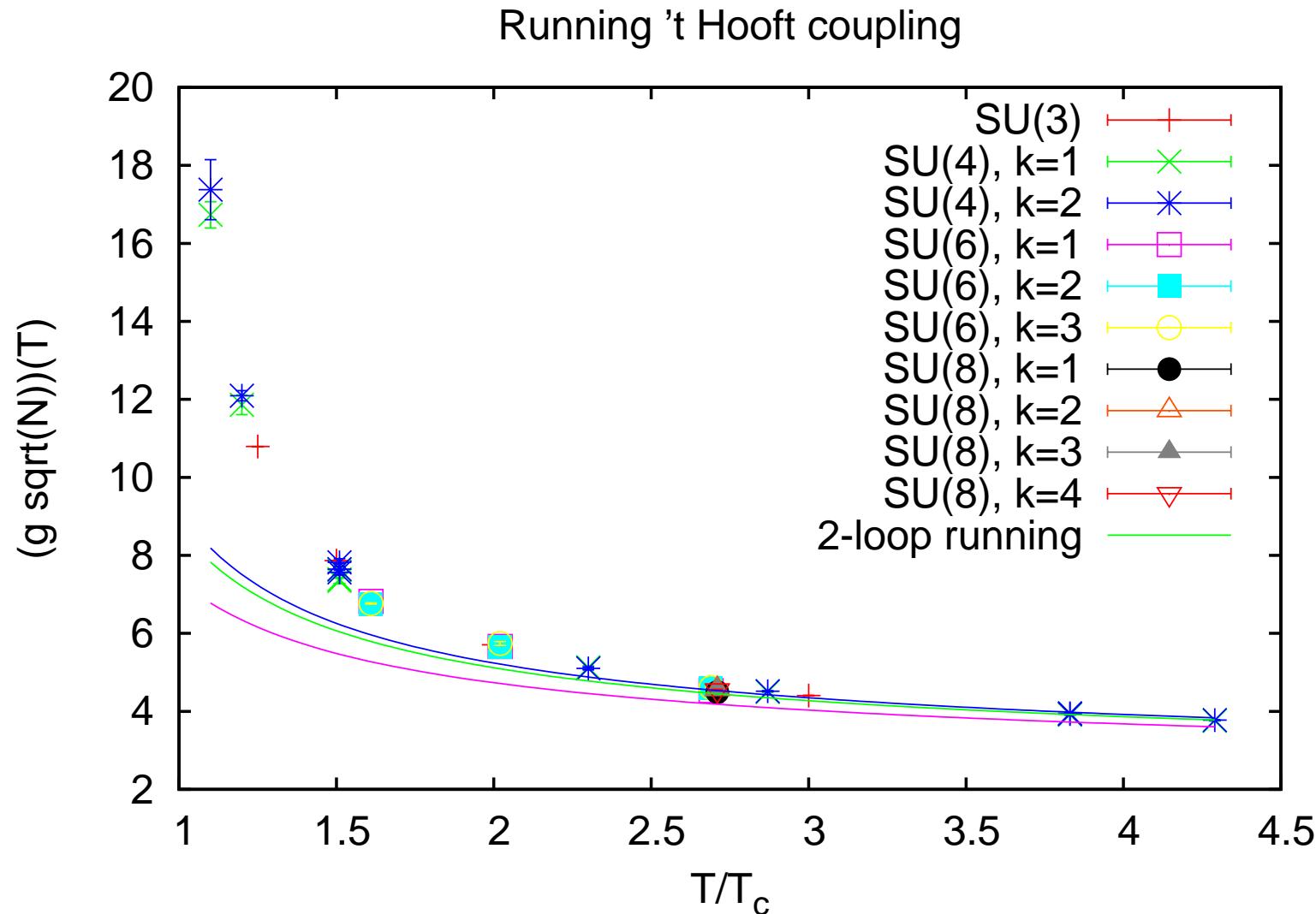
- Casimir scaling for  $T \gtrsim T_d$ .
- Far from perturbation theory.

Bursa and Teper '05

$SU(2, 3, 4, 6), T/T_d \geq 1.006, 20 \times 30 \times (4-5)$

- Same conclusions (but for  $\partial_\beta \Sigma_k$ )

**An interesting exercise** : get  $g(T)$  from  $\Sigma_k(T) = k(N - k) \frac{4\pi^2}{3\sqrt{3}} \frac{T^3}{\sqrt{g^2(T)N}}$ .



Put inside pressure formula : **Unfortunately, does not work.**

### III.C. Topology and instantons

**Strong 1st order** → calculate  $\langle \mathcal{O} \rangle$  in confined and deconfined phase at  $T_d$ .

→ interesting to calculate  $\chi \equiv \langle Q^2 \rangle / V$  with

$$\int d^4x \ Q(x) = \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \int d^4x \ \text{Tr} F^{\mu\nu}(x) F^{\rho\sigma}(x) \in \mathbb{Z}$$

**Instantons above  $T_d$ ?**  $D(\rho) \sim e^{-N \frac{8\pi^2}{g^2 N(\rho)}} \times e^{-N \rho^2 T^2}$  Gross, Pisarski, Yaffe '81

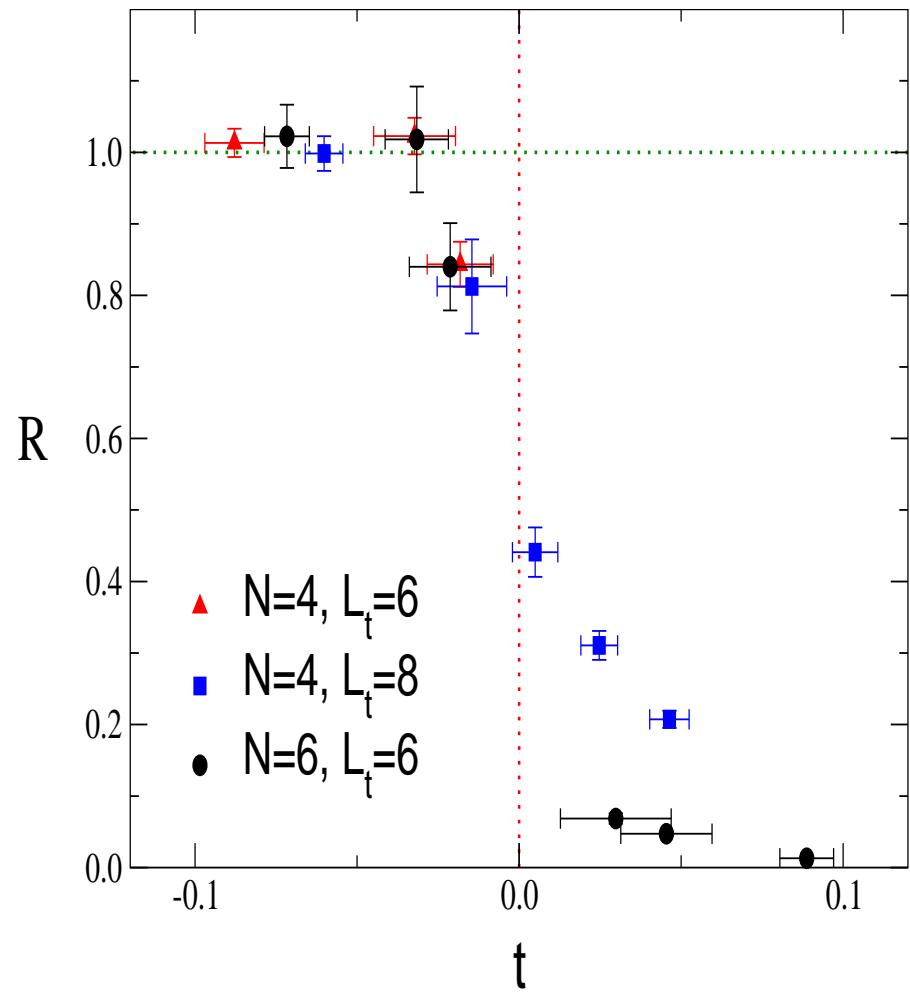
→ small and large instantons are suppressed (to  $e^{-N} \rightarrow 0$ )

→ leaves only  $\rho_{\min} < \rho < \rho_{\max}$ .

**This was checked on the lattice :**

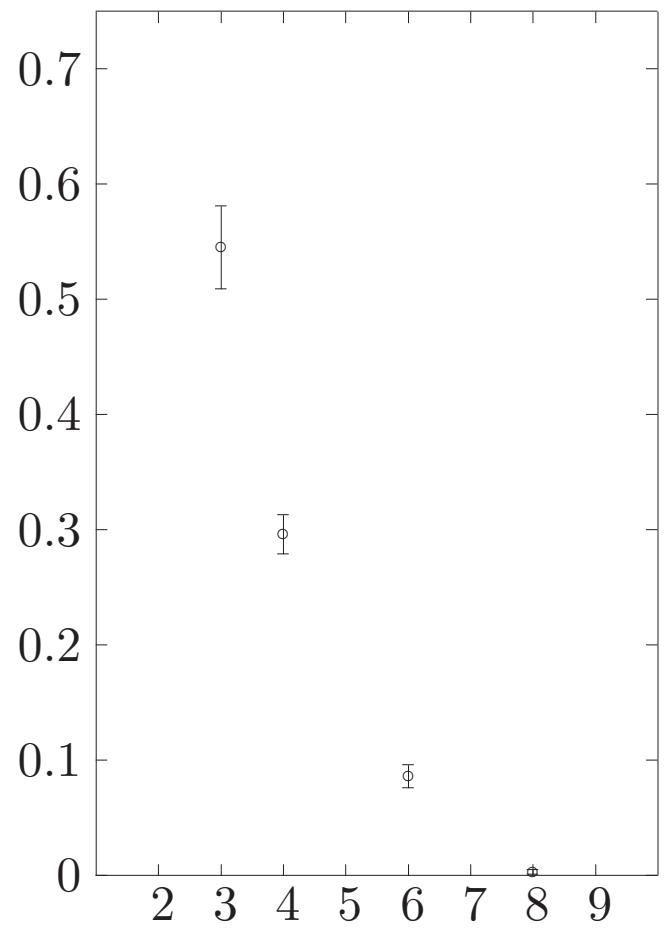
- Use “cooling” to smooth gauge configurations.
- Get  $\text{peak}[Q(x)] = \frac{6}{\pi^2 \rho^4} \rightarrow \text{get } D(\rho)$

## Results : Lucini, Teper, Wegner '04, Del Debbio, Panagopoulos, and Vicari '04



$SU(4, 6), L_t = 6, 8, L_s/L_t = 4$

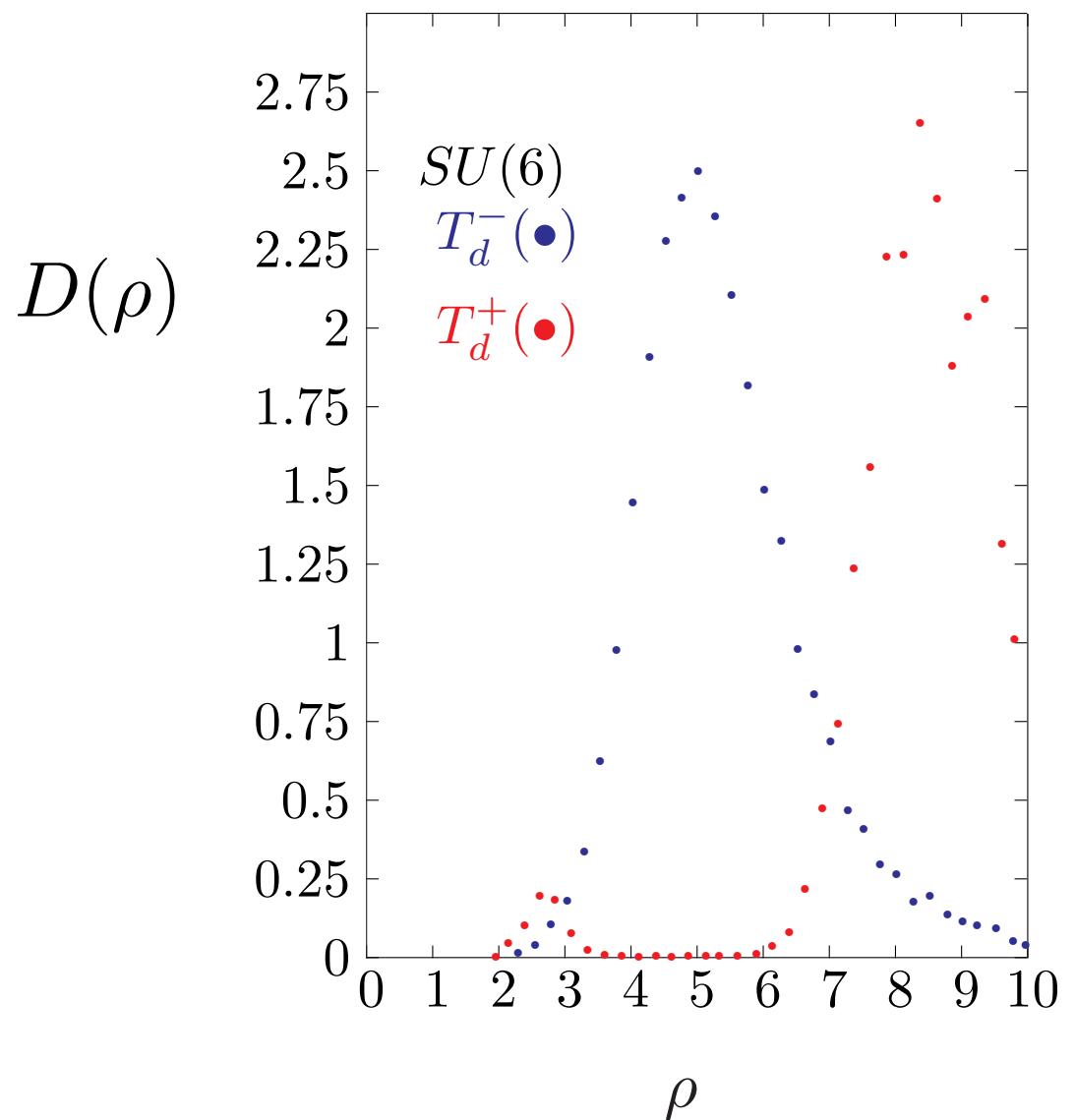
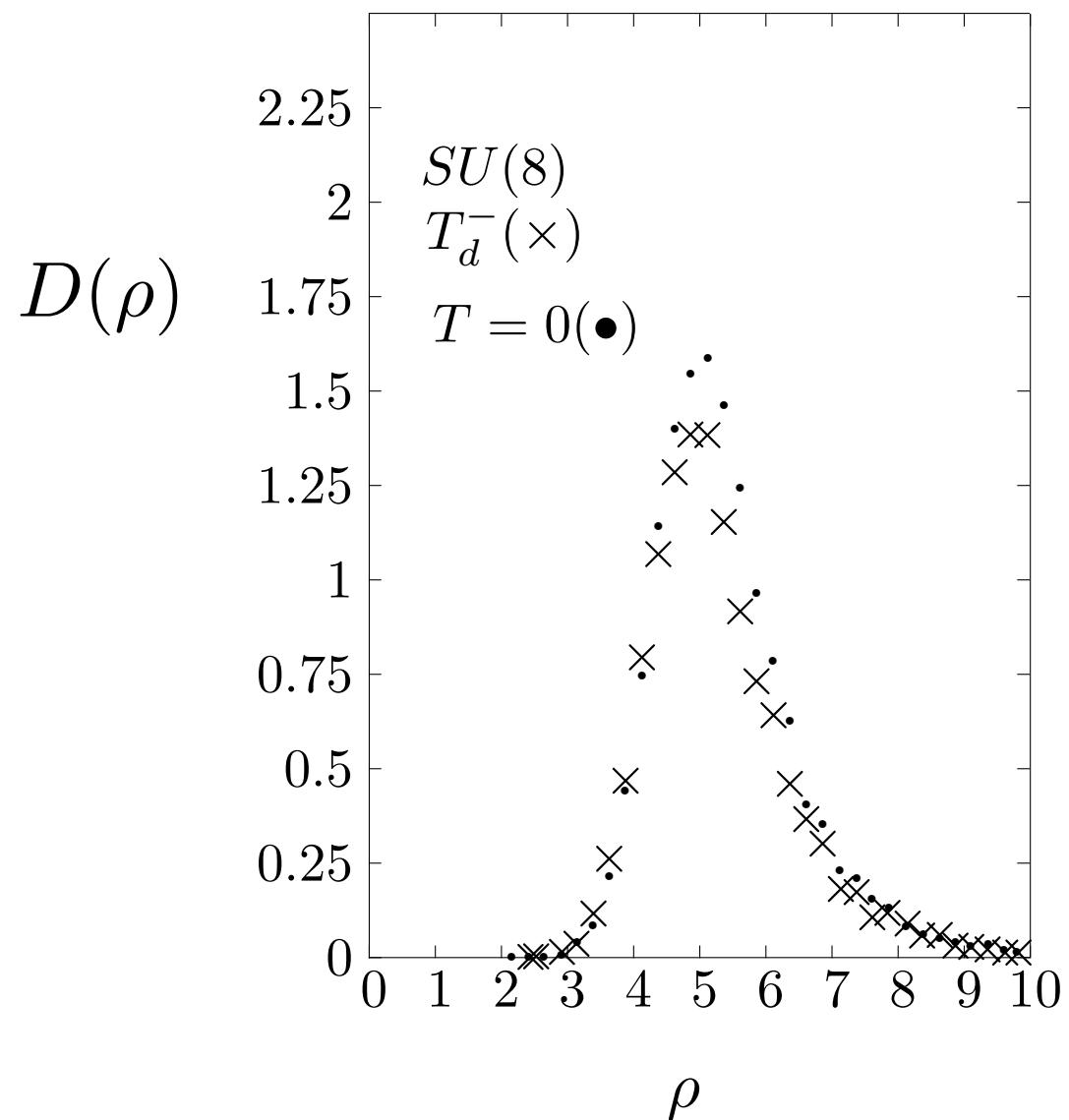
$$\frac{\chi_t^{decon}(T_c)}{\chi_t^{con}(T_c)}$$



$N$	$SU(3), 32^35,$	$SU(4), 32^35,$	$SU(6), 16^35,$	$SU(8), 12^35$
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Find that :  $\chi(0) = \chi(T_d^-); \chi(T_d^+) \sim e^{-N} \rightarrow 0 \Rightarrow \rho_{\max} < \rho_{\min}$  ? Kharzeev et al '98

**Check  $D(\rho)$  :**



**Similar features in other  $SU(N)$ .**

## IV. Summary

### What I covered

**Large  $N$  at  $T \simeq T_d$  :**

- Phase coexistence + finite size scaling.
- $L_h \simeq \frac{1}{2} S.B..$   $\implies$  1st order transition.
- $\xi < \infty$  ( $\rightarrow \infty$  at  $T_H \simeq 1.1T_d$ ).

**Large  $N$  at  $T > T_d$  :**

- “Pressure deficit” survives  $N = \infty$ .
- Mass gaps - “early” large- $T$  behavior ?
- Interface tensions - Casimir scaling but far from perturbations.

$$\bullet \quad \chi_{\text{topology}}^{\infty}(T) = \begin{cases} \chi_{\text{topology}}^{\infty}(0) & T < T_d \\ 0 & T \geq T_d \end{cases} .$$

## What I didn't cover but has been done

**Wetting phenomenon** and  $\sigma_{cd}$  vs.  $\Sigma_k$ .

**$SU(N \geq 4)$  @ 3D** 1st order: **Liddle and Teper '05-'06, Holland '05**

**Bulk thermo in 3D** : better approach to perturbation theory **Petersson et al. '06, Liddle '06**

**$T_d$  vs.  $T_\chi$** , vacuum alignment at  $1/N$ . **Narayanan and Neuberger '06**

## What can be (and some of it is being) done

**(Renormalized) Eigenvalue densities** vs. Polyakov loop actions **Dumitru et al. '04, Aharony et al. '04-'06**

**Pure gauge on a sphere** : **Aharony et al. '04-'06, BB and Wheater in progress.**

**Dual Hagedorn transition** : area laws for 't Hooft loops below  $T_d$ .

**Quarkonia at large- $N$**  : but first need  $T = 0$ .

**$\theta$  vacua at finite  $T$**  à la **Del Debbio, Panagopoulos and Vicari '04,'06**